VP160 FINAL REVIEW

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Angular Momentum

Rigid Body Dynamics

Equilibrium and Elasticity

Gravitation

Angular Momentum

Particle

$$\overline{L} = \overline{r} \times \overline{P}$$

$$\overline{L} = I \cdot \overline{\omega}$$

$$\overline{\tau} = \overline{r} \times \overline{F}$$

$$\overline{\tau} = \frac{d\overline{L}}{dt}$$

Angular Momentum

Rigid Body

$$\overline{L} = \sum_{i=1}^{N} m_i r_i \times (\omega \times r_i)$$

$$I = \sum_{i=1}^{n} m_i r_i^2$$

$$\overline{L} = I \cdot \overline{\omega}$$

Tensor Representation

$$\left[\overline{I}_{\alpha'\beta'} \right]_{\alpha',\beta'=x',y',z'} = \begin{bmatrix} \sum_{i=1}^{N} m_i (y_i'^2 + z_i'^2) & -\sum_{i=1}^{N} m_i x_i' y_i' & -\sum_{i=1}^{N} m_i x_i' z_i' \\ -\sum_{i=1}^{N} m_i y_i' x_i' & \sum_{i=1}^{N} m_i (x_i'^2 + y_i'^2) & -\sum_{i=1}^{N} m_i y_i' z_i' \\ -\sum_{i=1}^{N} m_i x_i' z_i' & -\sum_{i=1}^{N} m_i z_i' y_i' & \sum_{i=1}^{N} m_i (x_i'^2 + y_i'^2) \end{bmatrix}$$

Conservation of Angular Momentum

If the sum of all external torques on the system is equal to zero, then the total angular momentum of the system is constant (planar motion). The total angular momentum of a system can only be changed by external torques.

Energy in Rotation

$$E_k = \frac{1}{2} I_c \omega^2 + \frac{1}{2} m v_c^2$$

Parallel-axis Theorem(Steiner's Theorem)

$$I_O = I_C + m\overline{OC}^2$$

Perpendicular-axis theorem

$$I_x + I_y = I_z$$

Rolling without Slipping

$$\omega r_c = v_c$$

Rigid Body Dynamics

1. Newton’s Laws
2. Dynamics Laws for rotational motion
3. Conservation of energy
4. Conservation of momentum
5. Conservation of angular momentum

Equilibrium

$$F_{ext} = 0$$
$$\tau_{ext} = 0$$

Elasticity

$$elasticmodulus = \frac{stress}{strain}$$

Methods

1. Equilibrium equations.(Usually four)
2. Virtual work
3. Infinitesimal method
4. Derivation of energy

Gravitation

$$F = -G\frac{Mm}{R^2}$$

$$U = -G\frac{Mm}{R} + C$$

Kepler's Laws

1. Each planet moves in an elliptical orbit with the Sun at one of the focal points.
2. The line from the Sun to a given planet sweeps out equal areas in equal times.
3. T^2/a^3 is a constant.

Exercise 1

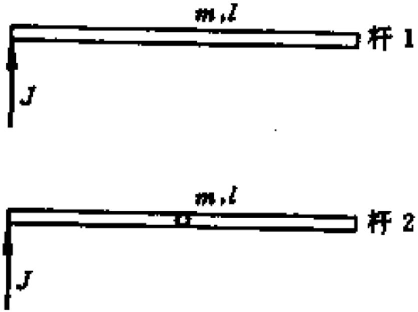
A cylindrical shaped pen was spinning around its axis at a constant angular velocity *omega* in the universe(no external force). At one moment, an instantaneous force was applied on one end of the pen, perpendicular to the axis. Discuss the motion of the pen later.

Exercise 2

1. Find the principle moment of inertia I of a stick with length l and mass m;
2. Find the principle moment of inertia I of a circle with radius r and mass m;
3. Find the principle moment of inertia I of a disk with radius r and mass m;
4. Find the principle moment of inertia I of a ball with radius r and mass m;
5. Find the principle moment of inertia I of a square with length of side a and mass m.
6. Find the moment of inertia I of a stick around one of its ends with length l and mass m;
7. Find the moment of inertia I of a disk around one of its diameter with radius r and mass m;

Exercise 3

Body 1 consists of one stick with mass m and length l. Body 2 consists of two sticks with mass $m/2$ and length $l/2$. The two sticks are connected with each other with a hinge. Apply an impulse J to these two bodies, as shown in the figure. Find E_1/E_2, where E_1 and E_2 are the energy of the two bodies respectively.



Exercise 4

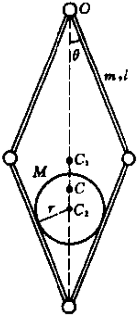
1. Discuss the motion of a particle that is placed on the inner surface of a spherical pot, close to its bottom, and released from hold (no friction).
2. Discuss the motion of a ball with radius r that is placed on the inner surface of a spherical pot with radius R, assume $R >> r$, close to its bottom, and released from hold (enough friction).

Exercise 5

A half cylinder is placed on the horizontal plane, and is covered by a strain with length πr and linear density λ. Find the tensile force of the strain at the top of the cylinder.

Exercise 6

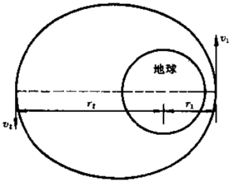
Find θ when the system is in static. Assume $l = 50\text{cm}$, $m = 50\text{g}$, $r = 8\text{cm}$, $M = 200\text{g}$.

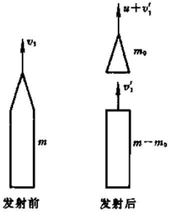


Exercise 7

As shown in the figure. The mass of airship is m, with a m_0 prober inside. At r_1, the velocity is $v_1 = \sqrt{\dfrac{2\alpha GM}{r_1}}$

1. Show that $1/2 < \alpha < 1$
2. At r_1, the prober is launched so that it moves in a parabola orbit. The airship than moves in a circular orbit. Find m_0/m and relative launch velocity u.
3. If launch the prober with the same velocity u, but at r_2 instead, find the orbit of the prober.





Exercise 8

As shown in the figure. The distance between the center of mass and the edge of table is $AC = b$. The stick is placed horizontally at the beginning and released from static. The friction coefficient is μ. Find θ when the stick begins to slide.